**Problem sheet 1: Tight-binding models, electrical transport** **12.10.2026**

### 1.1. Simple tight-binding model

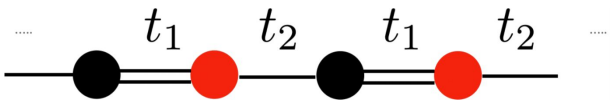
Consider atoms arranged on a square lattice:

(a) Write down the tight-binding expression for the band energy, $\varepsilon(\mathbf{k})$, assuming one orbital per site. The interactions run between nearest neighbors ($t_1 = 0.3\,\text{eV}$) and next-nearest neighbors ($t_2 = 0.15\,\text{eV}$).

(b) Sketch $\varepsilon(\mathbf{k})$ along the $\Gamma - X - M - \Gamma$ pathway where $X = (\frac{1}{2}, 0)$ and $M = (\frac{1}{2}, \frac{1}{2})$ lie on the zone boundary. Determine the bandwidth.

### 1.2. 1D Rice-Mele model

Consider a chain with two types of atoms and two hoppings, $t_1$ and $t_2$, as shown in the figure. The on-site energies are $+\Delta$ for site 1 (black) and $-\Delta$ for site 2 (red).




(a) Construct the $2 \times 2$ matrix of the tight-binding Hamiltonian. By diagonalizing this matrix, determine the energy dispersion of the bands, $\varepsilon(k)$, and sketch them pictorially.

(b) What condition should $\Delta$, $t_1$, and $t_2$ satisfy in order to have a finite gap for every $k$ in the spectrum separating the two bands?

### 1.3. Resistivity and conductivity in 2D

Consider a 2D metal with contacts attached to its two sides, such that $j_x \neq 0$ and $j_y = 0$:




(a) Use $\boldsymbol{j} = \sigma\mathbf{E}$ to express $j_x$ and $j_y$ in terms of $E_x$ and $E_y$.

(b) Express $\rho_{xx}$ and $\rho_{xy}$ in terms of $\sigma_{xx}$ and $\sigma_{xy}$.

(c) Consider magnetic field $\mathbf{B}$ applied along $z$. Express $j_x$ and $j_y$ in the presence of $\mathbf{B}$ using semiclassical theory, $m_e\mathbf{v} = \mathbf{F}\tau$ where $\mathbf{v}$ is electron velocity and $\tau$ is the mean-free time. Determine $E_y/E_x$.

(d) Determine the Hall coefficient, $R_H = E_y/(j_x B)$.

(e) Consider now the relation between $j_x$, $E_x$, and $E_y$ from part (c) in the high-field limit, $\omega_c\tau \gg 1$. Analyze the expressions from part (b). You should find (counter-intuitively) that $\rho_{xx} \sim \sigma_{xx}$.

*Submit the written solution to Problem 1.2 in the class:*
*tutorial sessions on 21.10.26 at 15:15 and 22.10.26 at 11:15.*

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*Indicate the completion of the problems 1.1 and 1.3, in percent*

| | **1.1** | **1.2** | **1.3** |
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